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2000 J. Phys.: Condens. Matter 12 2351

(http://iopscience.iop.org/0953-8984/12/11/301)

J. Phys.: Condens. Matter **12** (2000) 2351–2358. Printed in the UK PII: S0953-8984(00)07634-7

The Floquet spectrum and dynamical localization of a two-electron system driven by an alternating-current field

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Received 8 September 1999

Abstract. We study the dynamics of a pair of electrons in a double-well structure driven by an alternating-current field. With the help of the Floquet formalism, we find that the Floquet states undergo a series of level crossings and avoided crossings which are found to correspond to fundamental changes of the dynamical behaviour of the system. In particular, two electrons initially localized in one of the wells are found to be localized in perpetuity at the level crossing. When the system parameters are chosen to be at the avoided crossing, quantum beats occur. Due to the interchange of the Floquet states at the avoided crossing, the dynamical localization can be built up little by little.

Quantum systems driven by strong time-dependent external fields have received considerable attention [1], both in experimental and theoretical physics. Recently, a very intriguing result in the study of the role of time-dependent driving on the coherent dynamical tunnelling between two locally stable wells has been discovered by Grossmann *et al* [2]. For the appropriate field parameters, tunnelling is coherently suppressed; i.e., a localized packet, prepared as a superposition of two degenerate Floquet states of the system, remains localized in perpetuity. This phenomenon is often called coherent destruction of tunnelling (CDT). The CDT phenomenon is related to the exact crossing of two quasienergies of the double-well system and can be approximated by a two-level model if only the lowest doublet is involved [3]. Such field-induced tunnelling suppression has been shown to exist in H_2^+ and can be interpreted as field-induced dynamical localization [4]. In practical quantum many-body systems, the strong Coulomb interaction between the electrons cannot be ignored. For example, it has been verified that in mesoscopic systems the Coulomb interaction plays an important role in the quantum transport of the systems [5, 6]. So it is necessary to include the Coulomb interaction in investigating the dynamical behaviour of a quantum many-body system driven by a time-dependent field.

In the present work we investigate the time evolution problem for two electrons confined to a two-well structure and driven by an AC field. We construct an appropriate many-body basis so as to reduce the system Hamiltonian to a 3×3 matrix. By diagonalizing numerically the time evolution operator we obtain the quasienergies and Floquet states. With increasing Coulomb interaction we find that the Floquet states undergo a series of level crossings and avoided crossings. Qualitative changes of the dynamical behaviour of the system occur at these crossings. Although the Coulomb interaction between two electrons is very strong, we can control and suppress oscillation of the electron number in one of the wells. In

0953-8984/00/112351+08$30.00

particular, complete dynamical localization can be built up at the level crossing. With the system parameters chosen to correspond to the avoided crossing, the time evolution of the electron number in one of the wells shows conspicuous quantum beats. These results shed light on the nonlinear behaviour of quantum many-body systems exposed to time-dependent fields.

We suppose that there is just one energy level in each well. The Hamiltonian defining the system reads [7]

$$H(t) = \sum_{\sigma,k=L,R} \epsilon_k(t) C^+_{k\sigma} C_{k\sigma} + \widetilde{T} \sum_{\sigma} (C^+_{R\sigma} C_{L\sigma} + \text{h.c.}) + U(n_{L\uparrow} n_{L\downarrow} + n_{R\uparrow} n_{R\downarrow}) + W \sum_{\sigma_1,\sigma_2} n_{L\sigma_1} n_{R\sigma_2}. \tag{1}$$

Here, $C^+_{L\sigma}$ ($C^+_{R\sigma}$) creates an electron of spin σ in the left-hand (right-hand) well. If the time dependence is applied only to the wells, it will cause a proportionate shift in the energy levels:

$$\varepsilon_{L(R)}(t) = +(-)(\Delta\epsilon + V_0 \sin \omega t)/2$$

where V_0 is the amplitude of the AC field and ω the driving frequency with period T. $\widetilde{T}$ describes the coupling between two wells. U and W denote the intrawell and interwell Coulomb interaction, respectively. Because there are two electrons in the system, we can write the Hamiltonian in the space spanned by the basis vectors $|\bar{1}, \bar{1}\rangle$, $|11\rangle$, $|\bar{1}1\rangle$, $|1\bar{1}\rangle$, $|20\rangle$ and $|02\rangle$ where $|mn\rangle$ denotes the state of m electrons in the left-hand well and n electrons in the right-hand well, and the values 1 and $\bar{1}$ indicate the up spin and down spin, respectively [8]. Therefore the Hamiltonian (1) is described by a 6×6 matrix. If we replace the basis vectors $|\bar{1}1\rangle$ and $|1\bar{1}\rangle$ with $(|\bar{1}1\rangle - |1\bar{1}\rangle)/\sqrt{2}$ and $(|\bar{1}1\rangle + |1\bar{1}\rangle)/\sqrt{2}$, then the Hamiltonian can be written as follows:

$$H(t) = \begin{pmatrix} W I_{3\times 3} & 0 \\ 0 & H_1(t) \end{pmatrix} = W I_{3\times 3} \oplus H_1(t) \tag{2}$$

where $I_{3\times 3}$ is a 3×3 unit matrix and $H_1(t)$ is

$$H_1(t) = \begin{pmatrix} W & \sqrt{2}\widetilde{T} & \sqrt{2}\widetilde{T} \\ \sqrt{2}\widetilde{T} & U + 2\epsilon_L(t) & 0 \\ \sqrt{2}\widetilde{T} & 0 & U + 2\epsilon_R(t) \end{pmatrix}. \tag{3}$$

Obviously the many-body basis vectors $|\bar{1}\,\bar{1}\rangle$, $|11\rangle$ and $(|\bar{1}1\rangle - |1\bar{1}\rangle)/\sqrt{2}$ are the eigenvectors of the Hamiltonian and constitute the trivial triplet subspace, in which the electron number in each well is invariably one, and the time-dependent term does not influence this characteristic of the triplet subspace. Hence we will focus attention on the reduced Hamiltonian $H_1(t)$. For simplicity, in what follows we will ignore the detuning $\Delta\epsilon$ of the wells and the interwell Coulomb interaction W. We denote the basis vectors $(|\bar{1}1\rangle + |1\bar{1}\rangle)/\sqrt{2}$, $|20\rangle$ and $|02\rangle$ by $(1, 0, 0)^{\mathrm{T}}$, $(0, 1, 0)^{\mathrm{T}}$ and $(0, 0, 1)^{\mathrm{T}}$, respectively. In the absence of a time-dependent term, the unperturbed eigenvalues and eigenvectors (not normalized) of the Hamiltonian H_1 can be exactly solved:

$$\begin{aligned} &\varphi_1 = \{-a/\sqrt{2}\widetilde{T}, 1, 1\} && E_1 = b \\ &\varphi_2 = \{0, 1, -1\} && E_2 = U \\ &\varphi_3 = \{-b/\sqrt{2}\widetilde{T}, 1, 1\} && E_1 = a \end{aligned} \tag{4}$$

where we have defined

$$a = (\sqrt{U^2 + 16\widetilde{T}^2} + U)/2 \qquad b = (-\sqrt{U^2 + 16\widetilde{T}^2} + U)/2.$$

In general the intrawell Coulomb interaction U is much larger than $\widetilde{T}$. So the ground-state energy is much lower than the other two nearly degenerate levels. When the time-dependent term is involved, time periodicity of the Hamiltonian enables us to describe the quantum evolution process of the system in terms of the Floquet theory. Within the framework of the Floquet formalism we can reduce the problem of the solution of the periodically time-dependent Schrödinger equation to the determination of the one-period propagator $U(t, 0)$:

$$U(t, 0) = T\left\{\exp\left[-\mathrm{i}\int_0^t \mathrm{d}\tau\ H_1(\tau)\right]\right\} \qquad (\hbar = 1). \tag{5}$$

The Floquet states and quasienergies may be obtained by diagonalizing $U(T, 0)$. It is difficult to treat it exactly because $[H_1(t_1), H_1(t_2)] \neq 0$. Therefore using the initial condition $U(0, 0) = I_{3\times 3}$, we integrate numerically the equation

$$\mathrm{i}\frac{\partial}{\partial t}U(t, 0) = H_1(t)U(t, 0) \tag{6}$$

and diagonalize $U(T, 0)$ to obtain the quasienergies $\{\varepsilon_\alpha\}$ and the Floquet states $\{|u_\alpha(0)\rangle\}$. Given an initial state of the system $|\Psi(0)\rangle$, we have that its time evolution can be expressed in terms of the Floquet states as follows:

$$|\Psi(t)\rangle = \sum_\alpha \exp(-\mathrm{i}\varepsilon_\alpha t)|u_\alpha(t)\rangle\langle u_\alpha(0)|\Psi(0)\rangle. \tag{7}$$

It is apparent from (7) that only the Floquet states overlapping the initial wave function contribute to its subsequent time evolution. A quantity tailored to the dynamics of our model is the number of electrons occupying one of the wells:

$$N_{L(R)}(t) = \langle\Psi(t)|\sum_\sigma n_{L(R)\sigma}|\Psi(t)\rangle. \tag{8}$$

We will consider the time evolution of the system starting from initial state $|20\rangle$, i.e., two electrons are localized in the left-hand well at the beginning of the evolution. If the driving force is not applied to the system, obviously due to the strong Coulomb repulsion the electron number $N_L(t)$ in the left-hand well will oscillate between 0 and 2 in the subsequent time development. We will investigate the dynamical behaviour of the system for some special values of the driving amplitude V_0 and Coulomb interaction U. In particular we expect qualitative changes of the dynamical behaviour of the system to occur near crossings among the quasienergies. We have taken units such that $\omega = 1.0$, and for all calculations displayed in this paper $\widetilde{T} = 0.25$.

We present the Floquet spectrum versus Coulomb interaction U in figure 1(a). The value of the amplitude of the field chosen here is $V_0 = 1.15$. Three branches of quasienergies ε_1, ε_2 and ε_3 are indicated with open squares, closed squares and triangles, respectively. For $V_0 \to 0$, the quasienergies obey $\varepsilon_\alpha \to \varepsilon_\alpha^0 = E_\alpha \pmod\omega$. The enlargement of the region bounded by the rectangle in figure 1(a) is given in figure 1(b). It reveals that the quasienergies ε_1 and ε_3 form an exact crossing at the value of $U = 2.905\,84$. Although the quasienergies ε_1 and ε_2 approach each other, they do not intersect. In fact they form an avoided crossing with the centre at $U = 2.862\,99$.

To elucidate the effects of the level crossing and avoided crossing displayed in figure 1 on the quantum mechanical behaviour of the system we examine the dynamical evolution of an initial many-body state with the help of the function $N_L(t)$. Given that two electrons are initially in a localized many-body state $|20\rangle$, we therefore investigate the subsequent dynamical evolution of $N_L(t)$. We present in figure 2(a) and figure 2(b) the time evolution of the electron number in the left-hand well over the first 10^3 multiples of the driving period at the level crossing and avoided crossing shown in figure 1(b), respectively. We show in figure 2(a) that at the exact crossing of ε_1 and ε_3, the electron occupation number $N_L(nT)$ remains near 2 throughout

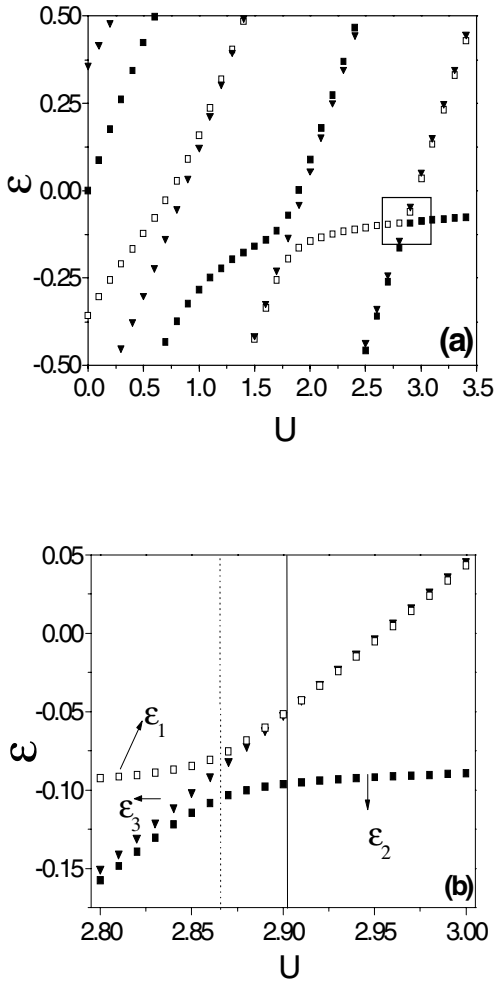

Figure 1. (a) The Floquet spectrum of the driven two-electron system as a function of the Coulomb interaction. (b) The small rectangular part of (a) is magnified to show the level crossing and avoided crossing. The vertical grid line and dotted line indicate the exact crossing and the centre of the avoided crossing, respectively.

the time development, as if two electrons are frozen in the left-hand well, although the strong Coulomb repulsion prevents them from being so. Moreover, time-resolved evolution over a few periods of the driving field (not shown) reveals that the two electrons also stay localized at times $t \neq nT$. We see in figure 2(b) that at the avoided crossing between ε_1 and ε_2 the dynamical behaviour of $N_L(nT)$ shows complex patterns, similar to quantum beats [2].

We note in figure 1(b) that on adiabatically increasing the value of the Coulomb interaction, an avoided level crossing occurs at first. If we continue increasing the Coulomb interaction, then the level crossing occurs, which corresponds to the complete dynamical localization shown in figure 2(a). This order of occurrence for the level crossing and avoided crossing in the parameter space of the system is essential for the existence of dynamical localization at the crossing of ε_1 and ε_3. To illustrate this characteristic, we present on the left-hand side of figure 3 the quasienergies versus U for five values of the amplitude of the field. The corresponding

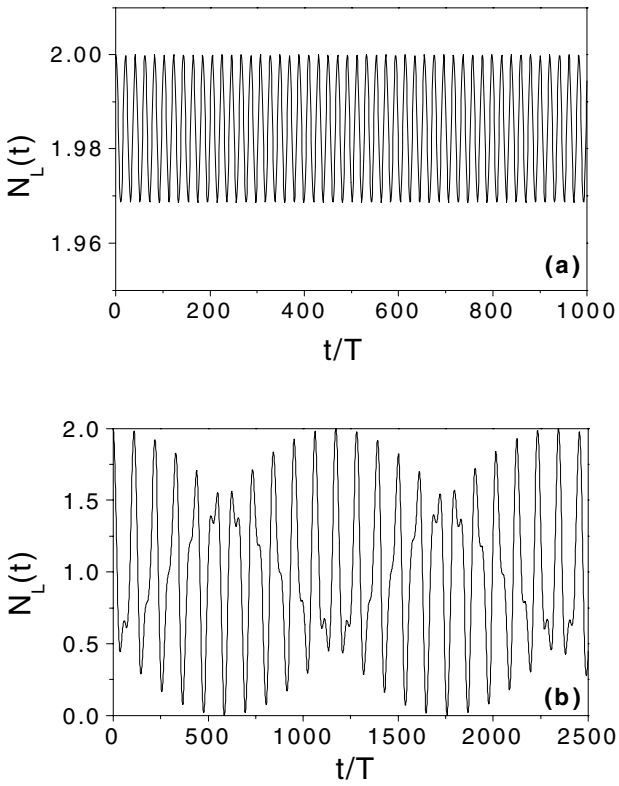

Figure 2. The time evolution of the electron number in the left-hand well over the first 10^3 multiples of the driving periods for (a) system parameters corresponding to the exact level crossing shown in figure 1(b), (b) the system parameters corresponding to the centre of the avoided crossing shown in figure 1(b).

time evolutions of the electron number functions $N_L(nT)$ at the exact level crossing are also displayed on the right-hand side of figure 3, with the same initial-state condition as was used for figure 2. We see in figure 3(a) that although the quasienergies ε_1 and ε_3 form an exact crossing at $U = 2.840\,58$, the phenomenon of dynamical localization ceases to exist at this crossing and the electron number in the left-hand well oscillates between 0 and 2. With increasing V_0, the level crossing and avoided crossing approach each other, and the corresponding time evolution of $N_L(nT)$ is frozen little by little. Finally, the two-level encounters exchange their order of occurrence on the Coulomb interaction axis and complete dynamical localization comes into being as shown in figure 3(e). So in figure 3 it is revealed that we can control, to any required degree, oscillations of the electron number in the left-hand well for appropriate system parameters.

This difference in dynamical behaviour at the crossings of ε_1 and ε_3 with different system parameters is ascribed to exchange of the state structure at the avoided crossing. For a driven two-level model it is well known that on adiabatically switching on the driving force, the adiabatic transition will occur, i.e., the two Floquet states exchange their structure at the avoided crossing. This pure quantum mechanical process also occurs in the present case and it can help build up the dynamical localization shown in figure 3(e). To illustrate this, we investigate the shapes of the Floquet states for six values of the Coulomb interaction chosen

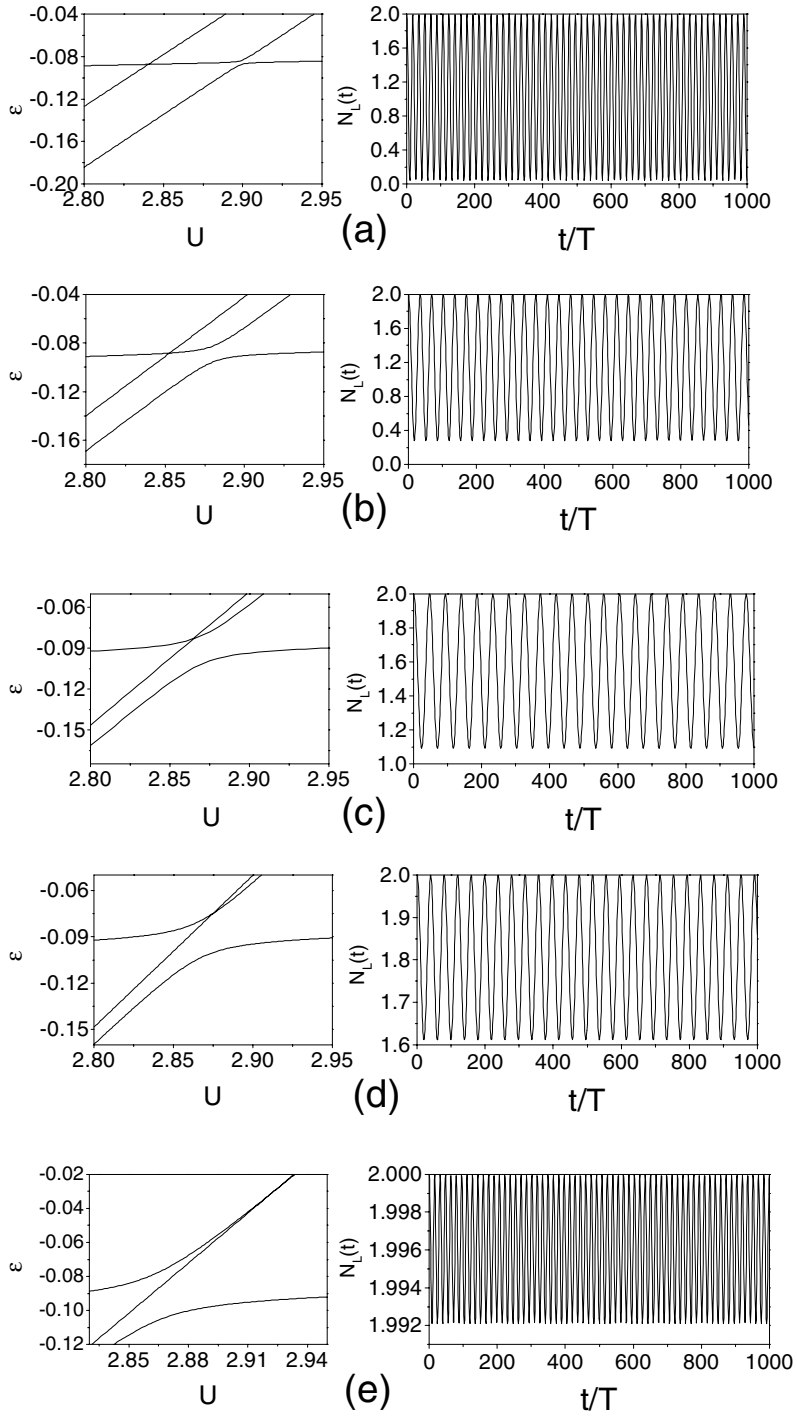

Figure 3. The Floquet spectrum of the driven two-electron system as a function of the Coulomb interaction and the corresponding time evolution of the electron number in the left-hand well over the first 10^3 multiples of the driving periods at the level crossing, for five values of the amplitude of the field: (a) $V_0 = 0.6$, where the value of U corresponding to the level crossing is $U = 2.840\,58$; (b) $V_0 = 0.9$, $U = 2.852\,81$; (c) $V_0 = 1.05$, $U = 2.864\,55$; (d) $V_0 = 1.1$, $U = 2.874\,78$; (e) $V_0 = 1.16$, $U = 2.921\,99$.

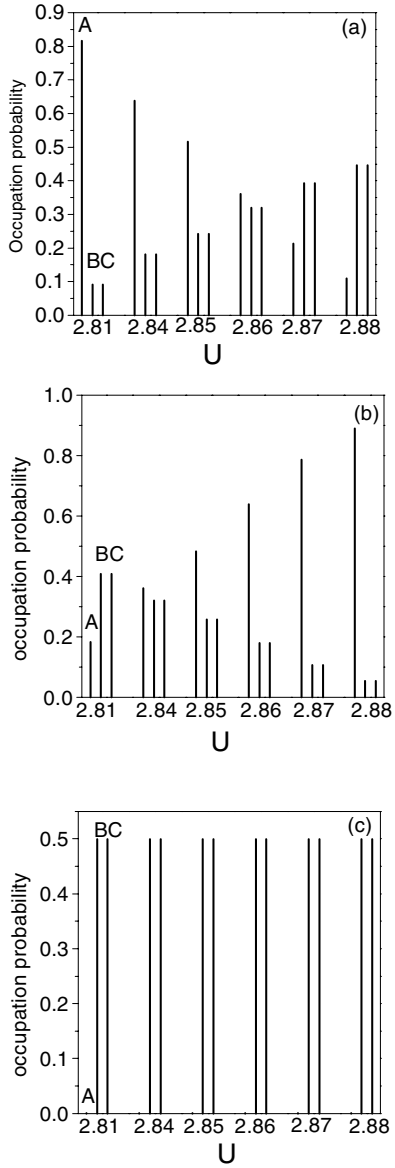

Figure 4. (a) The expansion of the Floquet state $|u_1(0)\rangle$ in terms of the many-body states A, B and C (see the text) for six values of the Coulomb interaction shown in figure 3(e). (b) The expansion of the Floquet state $|u_2(0)\rangle$. (c) The expansion of the Floquet state $|u_3(0)\rangle$.

around the avoided crossing shown on the left-hand side of figure 3(e). We present in figure 4 the occupation probability of the projection of all three Floquet states on the many-body states $(|\bar{1}1\rangle + |1\bar{1}\rangle)/\sqrt{2}$ (denoted by A), $|20\rangle$ (denoted by B) and $|02\rangle$ (denoted by C) for different values of U. The $|u_1(0)\rangle$, $|u_2(0)\rangle$ and $|u_3(0)$ states are presented in figure 4(a), figure 4(b) and figure 4(c), respectively. It is revealed in figure 4 that with increasing Coulomb interaction towards the centre of the avoided crossing (at $U = 2.86$) the occupation probability flows from state $|u_1(0)\rangle$ to $|u_2(0)\rangle$ and vice versa. The state $|u_3(0)\rangle$ is not affected by the avoided

crossing and its occupation probability remains constant. After exchanging their state structure, the states $|u_1(0)\rangle$ and $|u_2(0)\rangle$ turn into each other. Then, on further increasing the Coulomb interaction, the Floquet states $|u_2(0)\rangle$ and $|u_3(0)\rangle$ become degenerate at the exact level crossing. Because these two states are dominated by the many-body basis $|20\rangle$ and $|02\rangle$, at the level crossing the initial localized state $|20\rangle$ of the system can be approximated by the superposition of two degenerate Floquet states $|u_2(0)\rangle$ and $|u_3(0)\rangle$ that lead to complete suppression of the oscillation of the electron number in the left-hand well, as shown in figure 3(d).

In the above discussion and results, we do not consider the detuning $\Delta\epsilon$ of the wells. When $\Delta\epsilon$ is included, the spatial symmetry is broken by the detuning. We expect that an oscillatory beating behaviour will occur. Note that we have ignored higher-lying single-particle states; this requires the frequency of the external field to be much lower than the single-particle level spacing. If more than two interacting one-particle states are involved, the Pauli blockade, which is essential for the dynamical localization discussed above, will not be complete and thus the localization will be suppressed.

To summarize, we have numerically investigated the dynamical properties of a quantum many-body system driven by an AC field. The quasienergies and Floquet states are obtained by exact diagonalization of a whole-period time evolution operator. The Floquet states undergo a series of level crossings and avoided crossings. Qualitative changes of the dynamical behaviour occur at these crossings. In particular, two electrons initially localized in one of the wells can remain localized in perpetuity at the level crossing, even though the Coulomb repulsion between the electrons is very strong. We believe the present results to be useful in illustrating the dynamical properties of the driven quantum many-particle system.

Acknowledgments

This work was supported in part by the National Natural Science Foundation of China under Grant No 19725417.

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